ACCELERATION OF ELECTRICAL IMPEDANCE TOMOGRAPHY ALGORITHM WITH OPEN AND CLOSED DOMAIN MODEL EVALUATION

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Abstract: The presented paper discusses the possibilities of parallelizing algorithms for the image reconstruction in the Electrical Impedance Tomography (EIT). The parallelization process was performed and tested with combination of Matlab and CUDA environment. A twenty-fold acceleration of the Jacobian calculation was achieved. Besides of the parallelization process, the important part of the EIT experiments was the selection of the computational mesh. These mesh elements are used for fast execution of the partial differential equation EIT model. Forward and inverse problems of the image reconstruction were simulated on the novel open domain. The optimized domain reduced error in the reconstructed image.

Keywords: Electrical impedance tomography, open domain, parallelization, forward problem, inverse problem

1 INTRODUCTION

Electrical Impedance Tomography (EIT) is a non-destructive diagnostic method that can be employed in reconstructing the distribution of the impedance in an examined space. Principally, the method exploits an alternating current progressively applied to all the electrodes inserted on the border of the domain. The application of the alternating current enables us to measure the voltage on the electrodes that are not fed by the current source. The internal impedance is evaluated according to the measured voltage. The spatial resolution of the image is dependent on the number of connected electrodes [1].

EIT as a diagnostic method is applicable within multiple fields including: biomedicine, material engineering or geophysical mapping. In the biomedical engineering, the EIT is used for detection of the properties of tissue or the internal structure of the body. There exist two main ways for obtaining the results: the static and dynamic measurement of the voltage for image reconstruction. These properties enable the detection of the blood clots, tumors or lung oxidation during the breathing cycle. Regarding the material engineering, the discussed technique has major importance due to its non-destructive character. Based on this character, the EIT is powerful tool for monitoring heterogeneous particles, defects or cracks inside selected materials. The importance of EIT has also recently increased within geophysical mapping where this type of imaging is instrumental for detecting the formation of bedrock and subsoil or to observe groundwater and seepage in artificial lakes [1],[2],[3].

The process of EIT imaging includes a difficult numerical methods based on partial differential equation (PDE). For two dimensional space, the computation space is dependent on the triangular mesh structure. The quality of the mesh affects computation effort for image reconstruction. For a correct mesh design, the results of the simulation of the forward problem need to be independent on the selected mesh. This means that the different meshes are needed to be used for forward and inverse computation. The process of parallelization was applied on the iterative algorithm to reach an acceleration of the image reconstruction process.

2 THEORETICAL FRAMEWORK

EIT image reconstruction is a non-linear problem that combines the forward solution and non-linear inverse ill-posed problem with regularization. The Gauss-Newton method with the Tikhonov regularization was chosen for the image reconstruction. With the Tikhonov regularization, the objective function of the inverse problem assumes the form:

$$\Psi(\boldsymbol{\sigma}) = \frac{1}{2}\sum\nolimits_{\mathrm{ne}} \|\mathbf{U}_{\mathrm{M}} - \mathbf{U}_{\mathrm{FEM}}(\boldsymbol{\sigma})\|^2 + \alpha\|\mathbf{R}\boldsymbol{\sigma}\|^2, \tag{1}$$

where $\Psi(\boldsymbol{\sigma})$ is the regularized objective function, $\boldsymbol{\sigma}$ denotes the vector of the unknown distribution of conductivity in the monitored object, $\mathbf{U}_{\mathrm{M}}$ represents the vector of voltages measured on the electrodes, $\mathbf{U}_{\mathrm{FEM}}(\boldsymbol{\sigma})$ is the vector of voltage obtained via solving the forward problem, α stands for the regularization parameter, and $\mathbf{R}$ is the regularization matrix connecting neighbouring elements having different conductivities [1],[4].

The ill-posed problem is facilitated by the Tikhonov regularization that ensures a good convergence, stability and noise sensitivity reduction. The regularization matrix influence on the objective function is reduced with the increasing number of the iterations and also with the regularization parameter α [3].

Within the shown reconstruction, we seek such a vector $\boldsymbol{\sigma}$ that will minimize the objective function. In the given non-linear problem, minimization is most often performed via Gauss-Newton method. For the sought vector of conductivity $\boldsymbol{\sigma}$, the algorithm is defined by the expression:

$$\boldsymbol{\sigma}_{i+1} = \boldsymbol{\sigma}_i + \left(\mathbf{J}_i^{\mathrm{T}}\mathbf{J}_i + \alpha\mathbf{R}^{\mathrm{T}}\mathbf{R}\right)^{-1}\left(\mathbf{J}_i^{\mathrm{T}}\left(\mathbf{U}_{\mathrm{M}} - \mathbf{U}_{\mathrm{FEM}}(\boldsymbol{\sigma})\right) - \alpha\mathbf{R}^{\mathrm{T}}\mathbf{R}\boldsymbol{\sigma}_i\right), \tag{2}$$

where $\boldsymbol{\sigma}_{i+1}$ is the novel conductivity approximation, $\boldsymbol{\sigma}_i$ denotes the conductivity vector from the previous step, and $\mathbf{J}_i$ represents the Jacobian expressing the sensitivity of the electrode potentials to a change of the conductivity in the given element.[4]

3 ALGORITHM DESCRIPTION AND PARALLELIZATION

This section discusses the image reconstruction process using Gauss-Newton method with Tikhonov regularization. The relevant algorithm was designed in Matlab with the options CPU sequential processing or GPU-based partial parallelization [5].

The process of image reconstruction algorithm can be divided into three main parts: initialization, iterative calculation and show results function. In the initialization part, the selected mesh is loaded into the program environment where the number of nodes and elements and the number and position of the electrodes and inhomogeneous particles inside domain are extracted. The iterative calculation part starts with calculation of the regularization matrix. After that, the finite element method based computation is performed on initial data to gain the measured voltages on the electrodes (forward solution). We then compute the Jacobian that will be used in the subsequent solution of the Gauss-Newton iteration method. This step gave us a vector of conductivity for every iteration of the procedure. At the end of the iteration cycle, the regularization parameter α is lowered. The computation is limited by the count of iterations which equals 150. After the image reconstruction has been computed, the show results function is called for creating the representation of the impedance distribution in the simulated electrode system.

In this image reconstruction process, the most difficult phase consists in computing the Jacobian which is required for Gauss-Newton iteration algorithm. For this reason the novel data processing was prepared and substituted with the existing code. The novel Jacobian code is based on the CUDA platform run on the NVIDIA GPU device with using the VRAM. The CUDA function solve the computation in parallel. The difference in the resulting implementation is shown in the corresponding flowchart, Fig. 1.

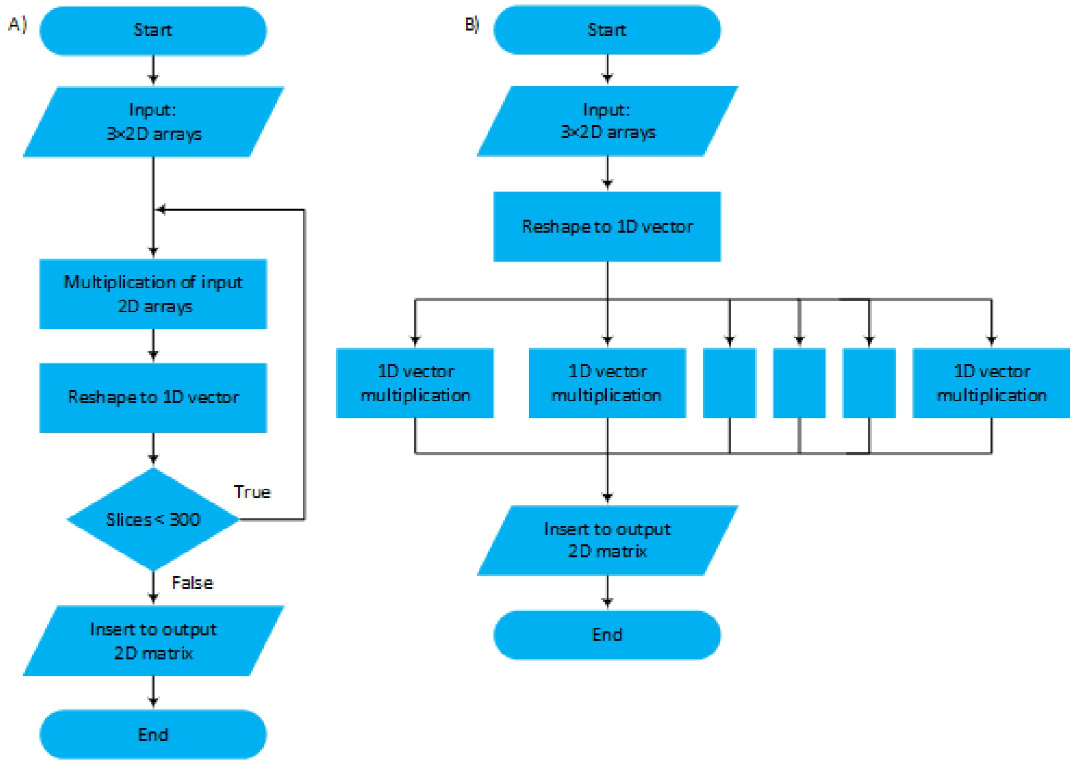

Figure 1: The flowchart diagram of the computing the Jacobian sequentially and in parallel

The Jacobian matrix had more than 8.3 million elements. The novel solution cut this matrix to slices that were distributed vectorally between 300 threads. Every thread is employed on one column of the resulting Jacobian matrix. All these loops are nested in the loop with 300 iterations. Another difference is the shifting conversion operation to the 1D vector before multiplication. The proposed solution was tested with the identical models. The computational times are shown in Tab 1 bellow.

Hardware	Time [ms]
CPU: Intel Core i5-4460 (3.2 GHz; x64;8GB RAM)	5
GPU: NVIDIA GTX 970 (1.215 GHz; 4GB GDDR5)	0.25

Table 1: Computing the Jacobian via the parallel and sequential processing

The measured data indicated that the GPU-based parallel process with CUDA function is twenty times faster in comparison to the sequential computing.

4 SIMULATION WITH OPEN AND CLOSED DOMAINS

This section discusses the second practical part that is focused on the simulation in the EIDORS libraries with using open and closed domains. The Matlab function was created for GMSH model generation to construct specific models. This gave us possibility to construct specific models. One of these specific models is the open domain that can simulate the real environment (artificial dams) without additional border inside as is often used (Fig. 2). The additional borders inside the domain causes the higher speed of the field expansion on the created border and the normal distribution of probability is violated. For this reason, the open domain model should have to be more accurate with comparison to the closed domain [6],[7].

The initial mesh models with inserted inhomogeneities were shown in the Figure 3. The inhomogeneous particles had sharp border. The initial model contains the mesh refinements near the electrodes for precision field discretization. This enables us to gain accurate result from forward solver.

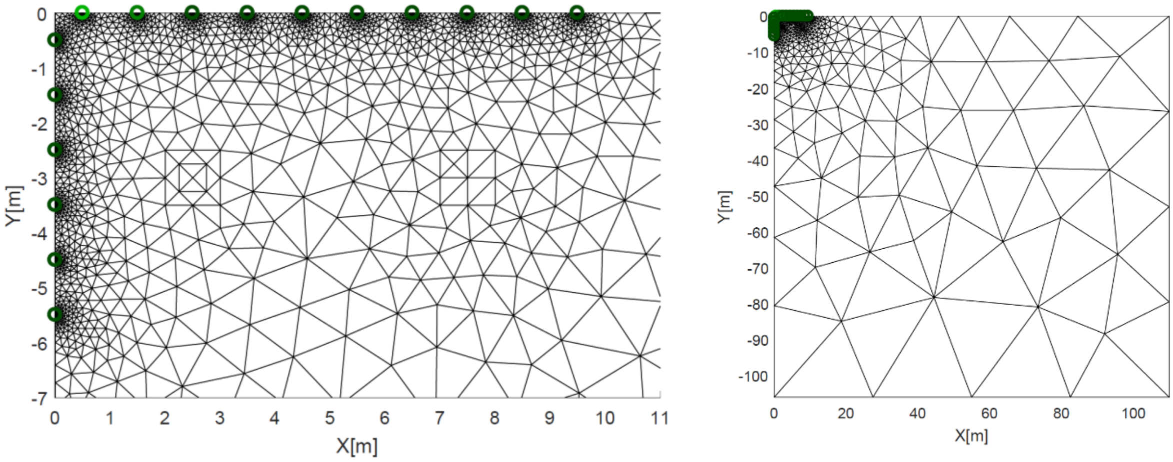

Figure 2: Forward unstructured mesh for open domain, electrodes are shown as circles. The left side shows detail of open domain, on the right side is displayed open domain completely

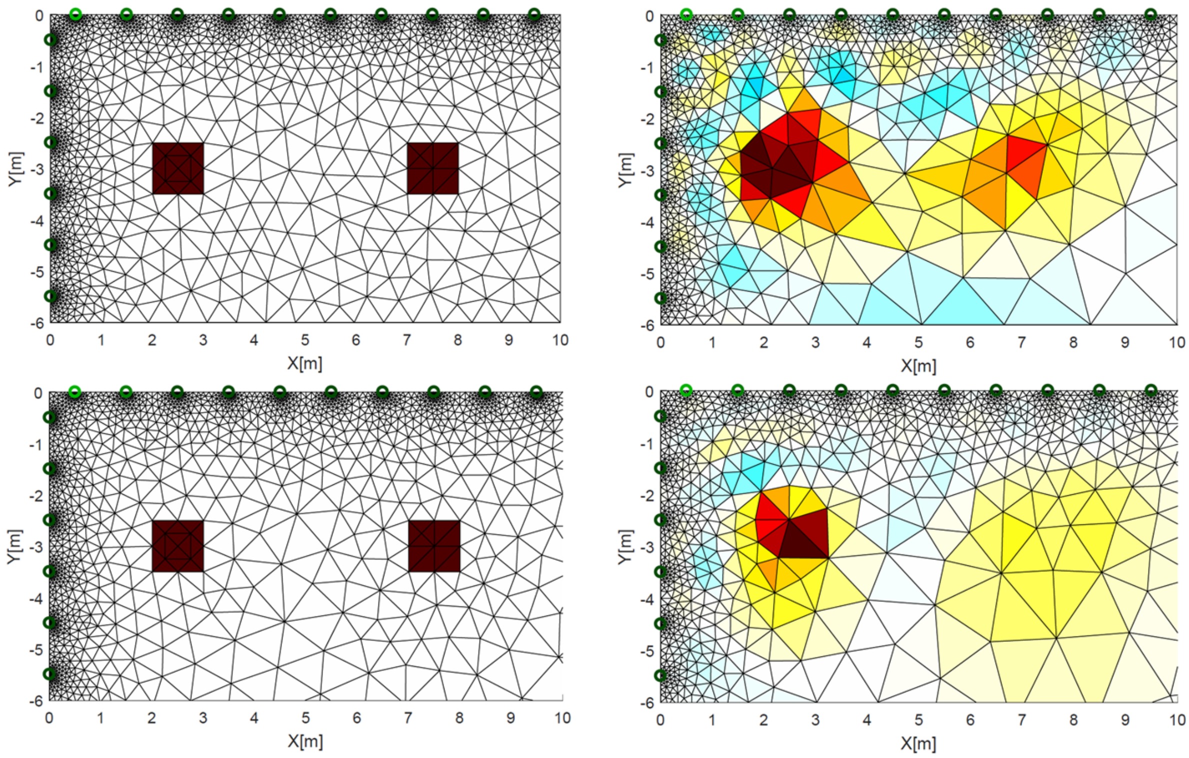

Figure 3: The top left picture shows the forward triangular mesh for closed domain, the top right figure shows the reconstructed image on the closed domain. The bottom left picture is the open domain model detail for forward computation, the bottom right picture shows the distribution of the impedance for open domain model. The Gauss-Newton method with Tikhonov regularization for simulation was used.

The results for closed and open domains were different. The closed domain reconstruction gave us good information about probability of area where the inhomogeneous objects were located. When we look on the open domain results, the one object inside domain was well detected. The second object, that is further from the electrodes, is detected but the maximum value is much lower in comparison to the closed domain. The difference between these two reconstructed images is given by the borders of the domain. In the case of the closed domain the field expands to the border and it is reflected back to the domain. This ping pong effect cause that the badly detected object is well reconstructed but it is not responding to the real environment. In the real environment (artificial dams) we do not have any additional borders underground and this is big difference. The open domain results gave as more realistic distribution of the impedance that should be compared in future with the reconstructed image from the real measurement.

5 CONCLUSION

This paper briefly describes the EIT image reconstruction process. The first part was focused on the mathematical formula of the Gauss-Newton method with Tikhonov regularization. In the practical part, the acceleration of the image reconstruction process was made successfully. The novel Jacobian computation using CUDA platform (Fig.1) with parallel threads was twenty times faster than sequential approach (Tab. 1). The second part was focused on the simulation results with closed and open domain mesh using EIDORS libraries. Two different models for the forward and inverse solver were created. The results gained from the specific closed and open domains with electrodes on two sides were evaluated. The comparison between the reconstructed impedance distributions gave us interesting information about diffusion and ping-pong effect that can affect the results in the incorrect way.

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